Probing the nature of the seesaw in renormalizable SO(10)

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Abstract

We study the nature of the see-saw mechanism in the context of renormalizable SO(10) with Higgs fields in the 10-plets and $\overline{126}$-plet representations, paying special attention to the supersymmetric case. We discuss analytically the situation for the second and third generations of fermions ignoring any CP violating phase. It is shown that $b-\tau$ unification and large atmospheric mixing angle strongly disfavor the dominance of the type I see-saw.

I. INTRODUCTION

We have shown recently [1] (see also [2]), by studying the second and third generations of fermions in the context of the minimal renormalizable SO(10) theory, that the so-called type II see-saw mechanism naturally connects $b-\tau$ unification with the large atmospheric mixing angle (θ_{atm}). Subsequent numerical studies for the full three generations case [3,4] further enhance the type II case and lead to an interesting prediction of a large 1-3 leptonic mixing angle, sitting on the experimental limit. Similar numerical studies [5,6] also show the same result for the type I see-saw, if one fine-tunes the CP phases (see however [7]). In this case, though, the connection between $b-\tau$ unification and large θ_{atm} is lost.

At this point one may believe that the issue is closed. However we think that analytical results are extremely important since they provide insight in this often obscure issue. This was the spirit of our original work [1] and this is what forced us to work with 2^{nd} and 3^{rd} generations only and to ignore CP violating phases. In [1] an important question remained unanswered: what about type I (canonical) see-saw? We show here how in the same context (SO(10) with renormalizable interactions only) the experimental facts of approximate $b-\tau$ unification and large θ_{atm} strongly disfavor the dominance of type I see-saw in the case of real couplings and vacuum expectation values (vevs).

Before proceeding with our analysis, we briefly review the grand unified theory in question and the types of the see-saw mechanisms present in minimal left-right symmetric theories (such as SO(10)) in general. For some recent reviews and more references see for example [8–11].

II. SEESAW IN SO(10)

The idea of SO(10) grand unification is rather old [12], and even the more specific and appealing supersymmetric grandunified SO(10) theory is with us since long time [13,14]. By now there are many versions of this theory, and even many "minimal" versions [15–17]. In what follows we stick to the renormalizable version only, in order to be able to have a predictive theory. Non-renormalizable operators depend on what lies beyond SO(10) and thus are not under control. It was demonstrated recently [18] that the minimal such renormalizable theory is based on three generations of matter superfields in the spinorial 16-dimensional representation and the Higgs superfields in 10, 126, $\overline{126}$ and 210 representations. The theory is minimal in the sense of simplicity of the Lagrangian (although not of the computations, see [19–21]) and having the least number of parameters, i.e. predictability.

This theory is especially simple and predictive in the Yukawa sector: only two sets of Yukawas with only 15 real components. Strictly speaking this is all we need, and what we mean by 'renormalizable SO(10)' is just a theory where the Yukawa are due to interactions with Higgs fields in the 10-plet and $\overline{126}$-plet representations. The results we are about to present are valid for any theory with the same Yukawa sector, no matter how complicated the heavy Higgs sector is. What remains undetermined, though, is the nature of the see-saw mechanism [22]. As is well known, in any renormalizable left-right symmetric theory, such as for example SO(10), there are two different sources of see-saw [23,24]. The first is the canonical one, called type I, which takes place through the necessary presence of heavy right-handed neutrinos. The right-handed neutrinos get their masses through the $SU(2)_R$ triplet Δ_R, and L-R symmetry implies the existence of its left-handed counterpart, the $SU(2)_L$ triplet Δ_L. In a generic ordinary field theory it can be shown [23,24] that $\langle\Delta_L\rangle \neq 0$ necessarily, whereas in supersymmetry the situation is more delicate [25,26]. Let us recall briefly the salient features.

The fact that $\langle\Delta_L\rangle \neq 0$ can be seen most eloquently by studying the one-loop tadpole for Δ_L with the type I seesaw mechanism: it is clearly divergent. In supersymmetry there is a compensating diagram which cancels precisely the divergence. In other words, there is no a priori argument against $\langle\Delta_L\rangle = 0$. The point is simple: $\langle\Delta_L\rangle \neq 0$ emerges from a potential term (in symbolic notation)

$$V = \Delta_L \Phi^2 \Delta_R + \ldots \, , \tag{1}$$

where Φ is the $SU(2)_L \times SU(2)_R$ bi-doublet field $\Phi(2,2)$ with $B-L=0$. At the cubic level there is no such term, so in supersymmetry (susy) without any extra fields the above interaction is absent and $\langle\Delta_L\rangle = 0$. In other words, in the minimal susy L-R or Pati-Salam theory one has the canonical, type I see-saw. In order to achieve the $\Delta_L\Phi^2\Delta_R$ interaction, one has two possibilities in supersymmetry:

(a) the presence of a $SU(2)_L \times SU(2)_R$ field $S(3,3)$, so that

$$W = \Delta_L \Delta_R S + S\Phi^2 + MS^2 \, . \tag{2}$$

After integrating out the (heavy) field S, (1) emerges.

(b) the presence of a $B-L$ carrying bi-doublets X and $\overline{X}$, and so

$$W = \Phi \Delta_L X + \Phi \Delta_R \overline{X} + M X \overline{X} \, . \tag{3}$$

Again, (1) is obtained by integrating out the heavy fields X and $\overline{X}$. Equivalently, X and $\overline{X}$ could pick up vevs, or, more precisely, the light doublets may contain linear combinations of Φ and X and $\overline{X}$ bi-doublets.

One type of minimal renormalizable SO(10) theory contains 54 and 45 superfields [27,28], another one a single 210 superfield. In the former case, 54 contains S, whereas in the latter case 210 contains $(2,2,10)$ and $(2,2,\overline{10})$ fields X and $\overline{X}$ in the Pati-Salam $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{SU}(4)_C$ notation. Thus, in either case type II see-saw is present together with the type I, and the resulting physical consequences become rather hard to decipher. Is there a way of disentangling the two sources of seesaw? Unfortunately, this question can be answered only within a specific model, there is not a general answer. We are inclined to favor the case of the model with 210, for the following reasons. First, this reduces the number of fields (compared with 54+45) and thus the number of parameters in the Higgs part of the superpotential. Second, and more important, through $10_H 210_H 126_H$ and $10_H 210_H \overline{126}_H$ couplings in the superpotential, the $(2,2,15)$ fields in 126 and $\overline{126}$ mix with the $(2,2,1)$ field in 10_H and thus pick up vevs of order M_Z [23,29]. This in turn implies a possibility of correct mass relations for the first and second generations of fermions a la Georgi-Jarlskog [30] without any further model building.

It has to be stressed however, that what follows does not depend on supersymmetry or on the specific model chosen, but only on the coupling of the matter 16_F to Higgs 10_H and $\overline{126}_H$, and on the assumption that the relevant bidoublets get a nonzero vev.

In short, the Yukawa superpotential is given by (in an obvious notation)

$$W = Y_{10} 16_F 16_F 10_H + Y_{126} 16_F 16_F \overline{126}_H \ . \tag{4}$$

The resulting mass matrices take the form

$$M_U = v_{10}^u Y_{10} + v_{126}^u Y_{126} \ , \tag{5}$$
$$M_D = v_{10}^d Y_{10} + v_{126}^d Y_{126} \ , \tag{6}$$
$$M_{\nu_D} = v_{10}^u Y_{10} - 3 v_{126}^u Y_{126} \ , \tag{7}$$
$$M_E = v_{10}^d Y_{10} - 3 v_{126}^d Y_{126} \ , \tag{8}$$
$$M_{\nu_R} = \langle \Delta_R \rangle Y_{126} \ , \tag{9}$$
$$M_{\nu_L} = \langle \Delta_L \rangle Y_{126} \ , \tag{10}$$

where M_{ν_D} stands for the neutrino Dirac Matrix, and M_{ν_R} and M_{ν_L} for direct Majorana mass matrices for right-handed and left-handed neutrinos, respectively.

From (5)-(10) the light neutrino masses, after integrating out the right-handed neutrinos become the mixture of the type I and type II see-saw

$$M_N = -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D} + M_{\nu_L} \ . \tag{11}$$

A priori, both terms are equally important, and the resulting analysis becomes messy. In order to simplify the issue, we have recently discussed analytically the case of only two generations, the second and the third one, assuming type II seesaw. As mentioned at the outset, the type II contribution connects naturally the large atmospheric mixing angle with the $b-\tau$ unification.

In this work we address the issue of the comparison of the two sources of seesaw in the same context.

A. Useful definitions

The fermion masses depend on the composition of the light higgs particles, which we parametrize in the following manner. Let us decompose the doublet mass matrix M_H as

$$U_H^T M_H D_H = M_H^d \ , \tag{12}$$

where the zero-modes Higgs fields are defined as

$$H_u = (U_H^\dagger H_{+1})_1 \ , \ H_d = (D_H^\dagger H_{-1})_1 \tag{13}$$

and the first two components of H_Y are the weak doublets in 10 and $\overline{126}$ with hypercharge Y. Thus the expressions of the vacuum expectation values are

$$v_{10}^X = X_{11}^H v^X \quad , \quad v_{126}^X = X_{21}^H v^X \quad , \quad X = u, d \quad . \tag{14}$$

For later use, let us define some derived quantities:

$$x = \frac{U_{21}^H D_{11}^H}{D_{21}^H U_{11}^H} \quad , \quad y = \frac{D_{11}^H}{U_{11}^H} \quad , \tag{15}$$

$$\alpha = \left(4 \frac{U_{11}^H D_{21}^H}{D_{11}^H}\right)^2 \frac{v_u^2}{\langle \Delta_L \rangle \langle \Delta_R \rangle} \ . \tag{16}$$

III. CAN WE TELL TYPE I FROM TYPE II?

Let us begin discussing an important issue: comparing the two forms of see-saw is manifestly model dependent, thus one needs a theory of fermion masses in order to address it. For example, in distinguishing between the two types of see-saw one tacitly assumes that they have different forms. However, it is possible, at least in principle, that they are essentially equivalent. Type II says that $M_N \propto M_{\nu_R}$, but if $M_{\nu_D} \propto M_{\nu_R}$, type I would give the same result. It appears as too much fine-tuning, but as can be seen from (5)-(10) it only requires $v_{10}^u \ll v_{126}^u$. If this were to work, it would mean that the whole issue of the nature of see-saw is not well defined. However, it can be shown not to work in this case. What happens is the following.

Using

$$(M_U, M_{\nu_D}) = v^u (Y_U, Y_{\nu_D}) \quad , \quad (M_D, M_E) = v^d (Y_D, Y_E) \quad , \tag{17}$$

in (5)-(8), one obtains a relation among the Yukawa couplings of the charged fermions:

$$Y_E = \frac{1}{1-x} \left[4 y Y_U - (3 + x) Y_D\right] \ . \tag{18}$$

We are interested in the limiting case $M_{\nu_D} \propto M_{\nu_R}$, i.e. $v_{10}^u = 0$, or, better $U_{11}^H = 0$. From (15) this means $x, y = \infty$ with finite x/y. (18) becomes

$$Y_E = -4 \left(\frac{y}{x}\right) Y_U + Y_D \ . \tag{19}$$

In the 2-generation case this represents 3 equations for only two unknowns, y/x and the rotation angle θ_D in $D^T E$, where the unitary matrices X are defined by $Y_X = X Y_X^d X^T$ (we will assume all model parameters real, which gives X orthogonal). We introduce

$$\epsilon_u = \frac{y_c}{y_t} \quad , \quad \epsilon_d = \frac{y_s}{y_b} \quad , \quad \epsilon_e = \frac{y_\mu}{y_\tau} \quad , \tag{20}$$

and take into account that experimentally $\theta_q, \epsilon_i = \mathcal{O}(\delta \approx 10^{-2})$. Now we want to see how all this fits into the equation for the atmospheric angle. We get it from $M_N \propto Y_d - Y_e$: first

$$\tan 2\theta_l = \frac{\sin 2\theta_D}{\frac{y_\tau - y_\mu}{y_b - y_s} - \cos 2\theta_D} \ , \tag{21}$$

and then using (19)

$$\tan 2\theta_l = \frac{\sin 2\theta_q}{\frac{(1-\epsilon_u)[(1-\epsilon)(1+\epsilon_e)-(1+\epsilon_d)]}{(1+\epsilon_u)(1-\epsilon_d)} - \cos 2\theta_q} \ . \tag{22}$$

The first term in the denominator is $\mathcal{O}(\delta)$, which gives the experimentally unacceptable small atmospheric mixing angle $\theta_l = -\theta_q + \mathcal{O}(\delta^2)$. It means thus, that in the minimal theory type I cannot mimic the type II see-saw: even if it gives $M_N \propto Y_D - Y_E$, the over-constrained system does not allow a large atmospheric solution.

There is an important lesson in this: in this theory type I and type II are truly different. However, it is clear that the nature of the see-saw mechanism is a meaningful question only if one has a theory of fermion masses, i.e. restricted Yukawa couplings, otherwise one cannot exclude that the other version can be made equivalent, meaning that we cannot tell type I from type II by neutrino masses alone.

IV. THE 2 GENERATION CASE (WARM-UP EXPLORATIONS)

To illustrate the discussion which follows, we start with a simple result for the canonical, type I term. Take the 2-3 generation case, and work in the approximation of zero second generation masses compared to the third one. It is a straightforward exercise to derive the connection between the leptonic (atmospheric) mixing angle θ_l and the quark $(b-c)$ mixing angle θ_q,

$$\tan 2\theta_l = \frac{\sin 2\theta_q}{2\sin^2\theta_q - 5/9} \ , \tag{23}$$

if you assume $b-\tau$ unification (recall that with (6) and (8) it is not automatic). Manifestly, a small V_{cb} ($\theta_q \to 0$ limit) implies a small θ_{atm} ($\theta_l \to 0$). Clearly, without a possible judicious choice of CP phases [5,6] in the 3-generation case (i.e. fine-tuning), the type I see-saw is generically strongly disfavored.

We can do better. Let us study the general case in (11), which incorporates both types of see-saw, while still keeping the vanishing second generation masses for the sake of illustration.

After some straightforward computational tedium, one can derive a simple but eloquent formula which connects the leptonic and quark mixing angles

$$\tan 2\theta_l = \frac{\sin 2\theta_q}{2\sin^2\theta_q - \Delta} \,, \tag{24}$$

with

$$\Delta = \frac{1}{1-9\alpha}\left[-5\alpha + \epsilon\,(1-4\alpha)\right] \,, \tag{25}$$

where

$$\epsilon = \frac{y_b - y_\tau}{y_b} \tag{26}$$

and α is a relative measure of type I versus type II see-saw, defined in terms of the model parameters in (16). In the limit $\alpha \to \infty$ (type I) (23) is reproduced correctly: $\Delta = 5/9$ for $y_b = y_\tau$, and so in this approximation type I see-saw gives a wrong result of a small θ_{atm}. As we mentioned before, it is possible to fine-tune the CP phases in the full 3 generations case, but generically speaking there is a problem. On the contrary, in the $\alpha = 0$ limit (type II), $\Delta \approx 1 - y_\tau/y_b$ and the large atmospheric mixing is related to the $b-\tau$ unification at the GUT scale.

The type II see-saw emerges in the limit of large $\langle\Delta_L\rangle$, i.e. when $\langle\Delta_L\rangle \gg v_u^2/\langle\Delta_R\rangle$, whereas the type I dominates in the opposite case of small $\langle\Delta_L\rangle$. This much is obvious, and formulae (16) and (12) give in principle a way of quantifying this. Again, in principle, in a complete theory, such as the minimal SO(10), it may be possible to determine the nature of the see-saw mechanism from the first principles, but in practice it is hard. We have unification constraints, and the doublet-triplet splitting together with the $d = 5$ proton decay limits, and this can shed some light in the issue and will be studied in future. Here we opt for the bottom-up approach which as we have seen may help deciding the nature of the see-saw.

In this work, we focus on the masses of the second and third generations, for which we can show explicit analytical results. It is natural to expect that the large atmospheric neutrino mixing arises at this level of approximation (while other neutrino properties could require to include the first generation masses). This is why we will address the question on the nature of the seesaw taking advantage on the observed large atmospheric mixing angle θ_{atm}.

V. THE 2 GENERATION CASE (EXACT RESULTS, NO PHASES)

At this point one would like to be sure, that the approximation with massless second generation quarks and charged leptons is at least qualitatively correct. We will now give the general formulae and check the conclusions given before. We will see that the above approximation is not really needed for the type II see-saw to be strongly preferred by data.

A. General formulae

Let us start with two matrix equations, which are valid for any number of generations as well as for general, complex, parameters.

$$Y_E = \frac{1}{1-x}\left[4yY_U - (3+x)\,Y_D\right] \ , \tag{27}$$

$$Y_N = -\alpha \left[\frac{3\,(1-x)\,Y_D + (1+3x)\,Y_E}{4}\right](Y_D - Y_E)^{-1}\left[\frac{3\,(1-x)\,Y_D + (1+3x)\,Y_E}{4}\right] + (Y_D - Y_E) \ , \tag{28}$$

where

$$Y_N = \frac{4D^H_{21}}{\langle\Delta_L\rangle} M_N \tag{29}$$

is a dimensionless matrix proportional to the light neutrino matrix (11). It is the most important quantity of this paper.

Any symmetric matrix Y_X gets diagonalized by a unitary matrix X as $(X = U, D, E, N)$

$$Y_X = X Y^d_X X^T \ . \tag{30}$$

After defining the unitary matrices

$$V_q = D^\dagger U \quad , \quad V_l = E^\dagger N \quad , \quad V_D = D^\dagger E \quad , \tag{31}$$

we get the equations

$$V_D Y^d_E V^T_D = \frac{1}{1-x}\left[4yV_q Y^d_U V^T_q - (3+x)\,Y^d_D\right] \ , \tag{32}$$

$$\begin{aligned} V_l Y^d_N V^T_l &= -\alpha \left[\frac{3\,(1-x)\,V^\dagger_D Y^d_D V^*_D + (1+3x)\,Y^d_E}{4}\right]\left(V^\dagger_D Y^d_D V^*_D - Y^d_E\right)^{-1} \\ &\quad \times \left[\frac{3\,(1-x)\,V^\dagger_D Y^d_D V^*_D + (1+3x)\,Y^d_E}{4}\right] + \left(V^\dagger_D Y^d_D V^*_D - Y^d_E\right) \\ &= \left[1 - \alpha\frac{9\,(1-x)^2}{16}\right]\left(V^\dagger_D Y^d_D V^*_D - Y^d_E\right) - \alpha\frac{3\,(1-x)}{2} Y^d_E \\ &\quad -\alpha Y^d_E \left(V^\dagger_D Y^d_D V^*_D - Y^d_E\right)^{-1} Y^d_E \ . \end{aligned} \tag{33}$$

It has to be noticed, that in the general complex case V_q is not the CKM matrix, since it contains all the phases, i.e. it has as a general $N_f \times N_f$ unitary matrix N_f^2 real parameters - $N_f(N_f-1)/2$ angles and $N_f(N_f+1)/2$ phases.

We will however consider only the case of real parameters, i.e. no CP violating phases. In this case V_q is the truncated CKM matrix for the second and third generation ($s_q = \sin\theta_q$, $c_q = \cos\theta_q$):

$$V_q = \begin{pmatrix} c_q & s_q \\ -s_q & c_q \end{pmatrix} \ , \tag{34}$$

and similarly ($s_D = \sin\theta_D$, $c_D = \cos\theta_D$):

$$V_D = \begin{pmatrix} c_D & s_D \\ -s_D & c_D \end{pmatrix} \ . \tag{35}$$

Solving (32) it is easy to get ($t_q = \tan\theta_q$, $t_D = \tan\theta_D$)

$$\frac{1-x}{4} = \frac{\epsilon_d\left(1+\epsilon_u t_q^2\right) - \left(\epsilon_u + t_q^2\right)}{\left[(1-\epsilon)\,\frac{1+\epsilon_e t_D^2}{1+t_D^2} - 1\right]\left(\epsilon_u + t_q^2\right) - \left[(1-\epsilon)\,\frac{\epsilon_e + t_D^2}{1+t_D^2} - \epsilon_d\right]\left(1+\epsilon_u t_q^2\right)} \tag{36}$$

and

$$\begin{aligned} t_D = {} & \frac{(1-\epsilon_e)\left[(\epsilon_u - \epsilon_d) + (1-\epsilon_u\epsilon_d)\,t_q^2\right]}{2\,(1-\epsilon_u)\,(1-\epsilon_e\epsilon_d)\,t_q} \times \\ & \times \left[1 + \sigma\left(1 - \frac{4\,(1-\epsilon_u)^2\,(1-\epsilon_e\epsilon_d)\,(\epsilon_e-\epsilon_d)\,t_q^2}{(1-\epsilon_e)^2\left[(\epsilon_u-\epsilon_d)+(1-\epsilon_u\epsilon_d)\,t_q^2\right]^2}\right)^{1/2}\right] \ , \quad \sigma = \pm 1 \ . \end{aligned} \tag{37}$$

Finally, the expression for the leptonic mixing angle is

$$\tan 2\theta_l = \frac{\sin 2\theta_D\left(1 + \frac{\alpha D_1}{1-\frac{9\alpha}{16}(1-x)^2}\right)}{\frac{(1-\epsilon)(1-\epsilon_e)}{(1-\epsilon_d)} - \cos 2\theta_D + \frac{\alpha D_2}{1-\frac{9\alpha}{16}(1-x)^2}} \ , \tag{38}$$

with

$$D_1 = \frac{\epsilon_e(1-\epsilon)^2}{\left[\frac{\epsilon_d + t_D^2}{1+t_D^2} - \epsilon_e(1-\epsilon)\right]\left[\frac{1+\epsilon_d t_D^2}{1+t_D^2} - (1-\epsilon)\right] - \frac{(1-\epsilon_d)^2 t_D^2}{(1+t_D^2)^2}} \ , \tag{39}$$

$$\begin{aligned} D_2 = {} & 3\frac{(1-x)}{2}\frac{(1-\epsilon_e)(1-\epsilon)}{(1-\epsilon_d)} \\ & + \frac{(1-\epsilon)^2}{(1-\epsilon_d)} \times \frac{\left[\frac{\epsilon_d + t_D^2}{1+t_D^2} - \epsilon_e(1-\epsilon)\right] - \epsilon_e^2\left[\frac{1+\epsilon_d t_D^2}{1+t_D^2} - (1-\epsilon)\right]}{\left[\frac{\epsilon_d + t_D^2}{1+t_D^2} - \epsilon_e(1-\epsilon)\right]\left[\frac{1+\epsilon_d t_D^2}{1+t_D^2} - (1-\epsilon)\right] - \frac{(1-\epsilon_d)^2 t_D^2}{(1+t_D^2)^2}} \ . \end{aligned} \tag{40}$$

Equations (37)-(40) are our main results. In order to illustrate their meaning we will solve them in leading order of the small parameter

$$\delta = \mathcal{O}(\epsilon_i, \theta_q) \ . \tag{41}$$

The general procedure is the following: choose one of the two solutions ($\sigma = -1$ or $\sigma = +1$) for t_D in (37), then use it in (36), (39), (40) and finally in (38).

B. First solution: $\sigma = -1$

Let's start with the solution with the lower sign:

$$t_D \approx \frac{\epsilon_e - \epsilon_d}{\epsilon_u - \epsilon_d} t_q = \mathcal{O}(\delta) \ , \tag{42}$$

$$\frac{1-x}{4} \approx \frac{(\epsilon_u - \epsilon_d)}{(\epsilon_e - \epsilon_d) + \epsilon(\epsilon_u - \epsilon_e)} = \mathcal{O}(1) \ , \tag{43}$$

$$D_1 \approx -\frac{\epsilon_e(1-\epsilon)^2}{[\epsilon_d - \epsilon_e(1-\epsilon)]\,\epsilon} = \mathcal{O}\left(\frac{1}{\epsilon}\right) \ , \tag{44}$$

$$D_2 \approx 6\frac{(\epsilon_u - \epsilon_d)(1-\epsilon)}{(\epsilon_e - \epsilon_d) + \epsilon(\epsilon_u - \epsilon_e)} + \frac{(1-\epsilon)^2}{\epsilon} = \mathcal{O}\left(\frac{1}{\epsilon}\right) \ . \tag{45}$$

It is not difficult to show that the only hope for θ_l to be large is that $|\alpha| \leq \mathcal{O}(\delta^2)$ *and* $|\epsilon| \leq \mathcal{O}(\delta)$. So in this case type I see-saw alone seems to be excluded, while $b-\tau$ unification looks also crucial. This solution is reminiscent of the original one [1] with small θ_D and $b-\tau$ unification needed.

C. Second solution: $\sigma = +1$

Let us now consider the second solution (37):

$$t_D \approx \frac{\epsilon_u - \epsilon_d}{t_q} = \mathcal{O}(1) \ , \tag{46}$$

$$\frac{1-x}{4} \approx \frac{(\epsilon_u - \epsilon_d)}{(1-\epsilon)s_D^2} = \mathcal{O}(\delta) \ , \tag{47}$$

$$D_1 \approx -\frac{\epsilon_e(1-\epsilon)}{s_D^2} = \mathcal{O}(\delta) \ , \tag{48}$$

$$D_2 \approx -(1-\epsilon) = \mathcal{O}(1) \ . \tag{49}$$

Again it is not difficult to show that for $|\alpha| \leq \mathcal{O}(1)$ the leptonic mixing angle is $\mathcal{O}(1)$, and can even be maximal for some fine-tuned value of α. In general however, pure type I see-saw ($\alpha \to \infty$) can be excluded since it gives a small $\theta_l = \mathcal{O}(\delta)$. This result is independent on the value of ϵ, i.e. on $b-\tau$ unification. In the limit $y_{s,\mu} = 0$ this solution goes to the $\theta_D = 0$ solution of [1], which was found out to be unphysical in the case of just two bidoublets (but the solution here is physical, first because here we have more than two Higgs bidoublets and second because the limit $y_{s,\mu} = 0$ itself is not physical).

We see thus that on top of the old solution, we have a new one with a bigger parameter space allowed ($|\alpha| \leq \mathcal{O}(1)$) and no need for $b-\tau$ unification.

D. Discussion

Whether type I or type II dominates can be seen also directly from the matrices. The starting point is the right-hand-side of (33), which looks like

$$-\alpha\begin{pmatrix}\delta & \delta/\epsilon\\ \delta/\epsilon & 1/\epsilon\end{pmatrix}+\begin{pmatrix}\delta & \delta\\ \delta & \epsilon\end{pmatrix} \tag{50}$$

in the $\sigma=-1$ case, and

$$-\alpha\begin{pmatrix}\delta^2 & \delta\\ \delta & 1\end{pmatrix}+\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} \tag{51}$$

in the $\sigma=+1$ case. Each element in the matrices gives just the order of magnitude and should be thus thought as multiplied with a coefficient of order one.

It is now immediately clear that in the $\sigma=-1$ case type II see-saw dominates (all the single matrix elements) if $|\alpha|<\mathcal{O}(\epsilon^2)$ and $|\epsilon|\leq\mathcal{O}(\delta)$, because then each element of the first (type I) matrix is smaller than the corresponding element of the second (type II) matrix. In the $\sigma=+1$ case, type II see-saw dominates as soon as $|\alpha|\leq\mathcal{O}(\delta)$. But even in the partially mixed case, i.e. when $\mathcal{O}(\delta)<|\alpha|\leq\mathcal{O}(1)$ the sum in (51) equals just the second term and gives a large ($\mathcal{O}(1)$) atmospheric mixing angle. And this is again the case with a large mixing angle, but also in the mixed type I plus type II case with $\alpha=\mathcal{O}(\delta^2)$ the angle is already large.

So to summarize, if type II see-saw dominates, then the atmospheric mixing angle is large in the $\sigma=+1$ case, but in the $\sigma=-1$ case this happens only if $b-\tau$ unification is realized. The opposite is not always true however: if the angle is large, type II does not need to completely dominate. For a range of α the 33 element of type I can be of the same order as the 33 element of type II and still give a large angle (again, $b-\tau$ unification has to hold if $\sigma=-1$).

VI. SUMMARY AND OUTLOOK

The see-saw mechanism, it is often said, provides a simple and natural rationale for the smallness of neutrino mass. The real issue, though, is how to test it experimentally. In principle the answer is that simple: find the right-handed neutrino or a heavy $SU(2)_L$ triplet, whatever be its source, and find the corresponding couplings with light neutrinos. The trouble, as we all know, is that we expect the see-saw scale to be large, above 10^{10} GeV or so, and thus we are left with indirect consequences only. This is reminiscent of any high energy physics, such as proton decay in GUTs.

In this paper we addressed the issue of the nature of the see-saw mechanism in a context of a well defined theory, with a predictive low energy effective theory: a renormalizable SO(10) theory, with higgs fields in the 10-plet and $\overline{126}$-plet representations. In view of theoretical motivations, we discussed in more detail supersymmetric models but the results are of more general validity. We studied analytically the issue of the second and third generations only. We had already noticed that the type II see-saw, based on the heavy $SU(2)_L$ triplet fits nicely: $b-\tau$ unification automatically implies a large atmospheric mixing angle.

Here we completed the program of investigation: we find that in no case type I can be a dominant source of neutrino masses, unless one fine-tunes the CP violating phases [5,6] (but see [6]). However, type I can be present and even compete with type II. More work will be needed before one can disentangle, if ever, the two sources of the see-saw mechanism.

What about including the CP phases in this analytic work? We have made an attempt to do that, but the amount of computational tedium wipes off any transparency. The point

is a number of unmeasured phases beyond δ_{CKM}, which cannot be rotated away at the large scale where one is utilizing the SO(10) structure. Some of these phases could be measured one day (?) in proton decay factories, but, today, this is simply science fiction.

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Note Added:

Yesterday, a new paper that considers related issue appeared [31]. Where it is possible to compare, their results agree with ours. They investigate numerically the 3×3 complex case and conclude that a renormalizable model cannot account for a positive ρ in V_{CKM}. We would like to point out that they assume (their eq. (5) and (6)) that the light Higgs doublets live only in 10 and $\overline{126}$, while in general this is not necessarily the case. It would be interesting also to see, whether the different possibilities for θ_D (our cases $\sigma = +1$ and $\sigma = -1$) have been taken into account and analyzed.

REFERENCES

[1] B. Bajc, G. Senjanović and F. Vissani, Phys. Rev. Lett. **90** (2003) 051802 [arXiv:hep-ph/0210207].

[2] B. Bajc, G. Senjanović and F. Vissani, arXiv:hep-ph/0110310.

[3] H. S. Goh, R. N. Mohapatra and S. P. Ng, Phys. Lett. B **570** (2003) 215 [arXiv:hep-ph/0303055].

[4] H. S. Goh, R. N. Mohapatra and S. P. Ng, Phys. Rev. D **68** (2003) 115008 [arXiv:hep-ph/0308197].

[5] K. Matsuda, Y. Koide, T. Fukuyama and H. Nishiura, Phys. Rev. D **65** (2002) 033008 [Erratum-ibid. D **65** (2002) 079904] [arXiv:hep-ph/0108202].

[6] T. Fukuyama and N. Okada, JHEP **0211** (2002) 011 [arXiv:hep-ph/0205066].

[7] K. Matsuda, arXiv:hep-ph/0401154.

[8] B. Bajc, A. Melfo and G. Senjanović, arXiv:hep-ph/0304051.

[9] B. Bajc, arXiv:hep-ph/0311214.

[10] R. N. Mohapatra, arXiv:hep-ph/0306016.

[11] S. Raby, arXiv:hep-ph/0401155.

[12] H. Georgi, *In Coral Gables 1979 Proceeding, Theory and experiments in high energy physics,* New York 1975, 329 and H. Fritzsch and P. Minkowski, Annals Phys. **93** (1975) 193.

[13] C. S. Aulakh and R. N. Mohapatra, Phys. Rev. D **28** (1983) 217.

[14] T. E. Clark, T. K. Kuo and N. Nakagawa, Phys. Lett. B **115** (1982) 26.

[15] K. S. Babu, J. C. Pati and F. Wilczek, Nucl. Phys. B **566** (2000) 33 [arXiv:hep-ph/9812538].

[16] T. Blažek, S. Raby and K. Tobe, Phys. Rev. D **60** (1999) 113001 [arXiv:hep-ph/9903340].

[17] C. H. Albright and S. M. Barr, Phys. Rev. Lett. **85** (2000) 244 [arXiv:hep-ph/0002155].

[18] C. S. Aulakh, B. Bajc, A. Melfo, G. Senjanović and F. Vissani, arXiv:hep-ph/0306242.

[19] C. S. Aulakh and A. Girdhar, arXiv:hep-ph/0204097.

[20] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, arXiv:hep-ph/0401213.

[21] B. Bajc, A. Melfo, G. Senjanović and F. Vissani, arXiv:hep-ph/0402122.

[22] T. Yanagida, proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan 1979 (edited by A. Sawada and A. Sugamoto, KEK Report No. 79-18, Tsukuba); S.L. Glashow, in *Quarks and Leptons*, Cargése, eds. M. Lévy, et al., (Plenum, 1980, New York), p. 707; M. Gell-Mann, P. Ramond and R. Slansky, proceedings of the Supergravity Stony Brook Workshop, New York, 1979, eds. P. Van Niewenhuizen and D. Freeman (North-Holland, Amsterdam); R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44** (1980) 912.

[23] G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181** (1981) 287.

[24] R. N. Mohapatra and G. Senjanović, Phys. Rev. D **23** (1981) 165.

[25] C. S. Aulakh, A. Melfo, A. Rašin and G. Senjanović, Phys. Lett. B **459** (1999) 557 [arXiv:hep-ph/9902409].

[26] A. Melfo and G. Senjanović, Phys. Rev. D **68** (2003) 035013 [arXiv:hep-ph/0302216].

[27] D. G. Lee and R. N. Mohapatra, Phys. Rev. D **51** (1995) 1353 [arXiv:hep-ph/9406328].

[28] C. S. Aulakh, B. Bajc, A. Melfo, A. Rašin and G. Senjanović, Nucl. Phys. B **597** (2001) 89 [arXiv:hep-ph/0004031].

[29] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70** (1993) 2845 [arXiv:hep-ph/9209215].

[30] H. Georgi and C. Jarlskog, Phys. Lett. B **86** (1979) 297.

[31] B. Dutta, Y. Mimura and R.N. Mohapatra, arXiv:hep-ph/0402113.